# Proof of Mean and Variance of Binomial distribution

## The very easy way

For the case where you get to do the experiment only once and there can be two outcomes Success with probability of p and failure with probability of (1-p) then the random variable is following a Bernoulli distribution.

So for Bernoulli distribution:

|  |  |
| --- | --- |
| **X** | **P(X)** |
| 1 | p |
| 0 | 1-p |

So the expected value of the random variable is defined as:

So for a random variable following a Bernoulli distribution, **E(X) = p\*1 + 0\*(1-p) = p.**

For a random variable:

And V(X) = E(X2) – {E(X)}2

So for a random variable following a Bernoulli distribution, **E(X2) = p\*12 + 02\*(1-p) = p.**

So for a random variable **V(X) = p - p2 = p(1-p).**

Now for a random variable following a Binomial distribution we are allowed to perform experiment n times. In other words, we are performing n Bernoulli experiments. Thus the Expected values are represented as:

* X = X1 + X2 + … + Xn
* E(X) = E(X1 + X2 + … + Xn)
* E(X) = E(X1) + E(X2) + … + E(Xn)
* **E(X) = np**
* V(X) = V(X1 + X2 + … + Xn)
* **V(X) = np(1 - p)**

## The very hard way

Link: https://www.youtube.com/watch?v=8fqkQRjcR1M

For binomial theorem if the probability of success = p and there are n trials, then the probability of getting x success is given as

=

Now we know that the expected value of a random variable x is given as E(x) =

For the binomial distribution the E(x) =

Since at x = 0 the equation in RHS is 0 we change the equation into

Now x! = x(x-1)(x-2)…2x1 = x(x-1)!

So we can cancel the x and change the equation as

Now we take n and p outside the summation block as below:

Now changing limits of the equations as below:

(n-x) = (n-1)-(x-1)

m = n-1

y = x-1

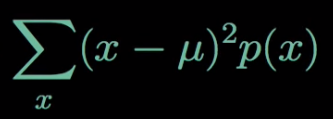
This gives rise to binomial theorem which is:

Where a = p b = 1 – p

So

Thus E(x) = np.

Now Variance V = E[(X - µ)2]

V = 

Also E[(X - µ)2] = E(X2) – [E(X)]2

We worked out that E(X) = np. So we have to find E(X2).

E(X2) =